Agent-Based Modeling of Networks of Logistic Maps with Long-Range Coupling

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Abstract

We present agent-based modeling (ABM) implementation of the model of the network of coupled map lattices (CML) with long-range coupling. We use Java version of the RePast ABM software package. Current report describes our preliminary results using the simplest network, namely 1D and 2D globally coupled logistic maps (GCM). Our agent-based model exhibits typical phenomena of the CML model, such as periodic dynamics, spatiotemporal chaos and pattern formation. We also discuss possible further development of the model.

Keywords: agent-based modeling, RePast, logistic map, long range interactions.

Abbreviations:
ABM – agent-based modeling,
CML – coupled map lattice,
EOC – edge of chaos,
GCM – globally coupled maps,
SMSD – standard mean square deviation.

The Model

We consider a network of N nodes and let each node of the network be assigned a dynamical variable x_n, n=1…N (see e.g. (Kaneko 1993; Amritkar et al. 2003; Antenedeo et al. 2004)). The evolution of the dynamical variable is given by

$$x_n(i+1) = (1-B)\,f(x_n(i)) + \frac{B}{\eta(C)}\sum_{\substack{m=1\\ m\neq n}}^{N}\frac{f(x_m(i))}{r_{nm}{}^{C}} \qquad (1)$$

where $x_n(i)$ is a dynamical variable of the *n*th node at the *i*th time step, *B* is the coupling strength between the nodes and r_{nm} is the distance between connected nodes *n* and *m* (in this case all nodes are globally coupled). The case when the power C is smaller than the dimension D corresponds to the long-range interactions (so-called "non-extensive" system), and the case when the power C is larger corresponds to the short-range interactions ("extensive" system), see e.g. (Anteneodo et al. 1998; Tamarit et al. 2000; Pluchino et al. 2004) for the discussion. The $\eta(C)$ is the normalization factor

$$\eta(C) = \sum_{\substack{m=1\\ m\neq n}}^{N}\frac{1}{r_{nm}{}^{C}} \qquad (2)$$

The function *f(x)* defines local dynamics ($x \to f(x,a)$) of the uncoupled individual node. In this paper, we define the local dynamics by the logistic map

$$f(x,a) = 1 - a x^2,\ x_n \in [-1:1],\ a \in [0:2] \qquad (3)$$

(this form of the map is isomorphic to the more "popular" form $f(x,a) = a x(1-x)$). For values of *a* smaller than $a_c = 1.40115519809...$, the map is regular and for values of *a* larger than a_c, the map is chaotic. The dynamics of the map at a_c corresponds to the so-called edge of chaos (EOC) regime (Fig. 1).

A convenient approach to study synchronization in the GCM is to compute the standard mean square deviation (SMSD) (see e.g. (Lind et al. 2004)):

$$\sigma_i^2 = \frac{1}{N}\sum_{n=1}^{N}\left(x_n(i) - < x(i) >\right)^2 \qquad (4)$$

Fully synchronized ("coherent") states correspond to the case when $\sigma_i^2 \to 0$.

In the present report we consider the simplest network, namely GCM on 1D and 2D lattice. When using ABM methodology, each map is represented by an agent, immobilized on the grid point. Each agent interacts with all other agents. The state of the agent is given by the value of the dynamical variable $x_n(i)$ and is defined using the rule based on Eq. 1.

We used Java edition of the RePast version 3 ABM software (http://repast.sourceforge.net). The Java source code was written using the Eclipse platform (http://www.eclipse.org).

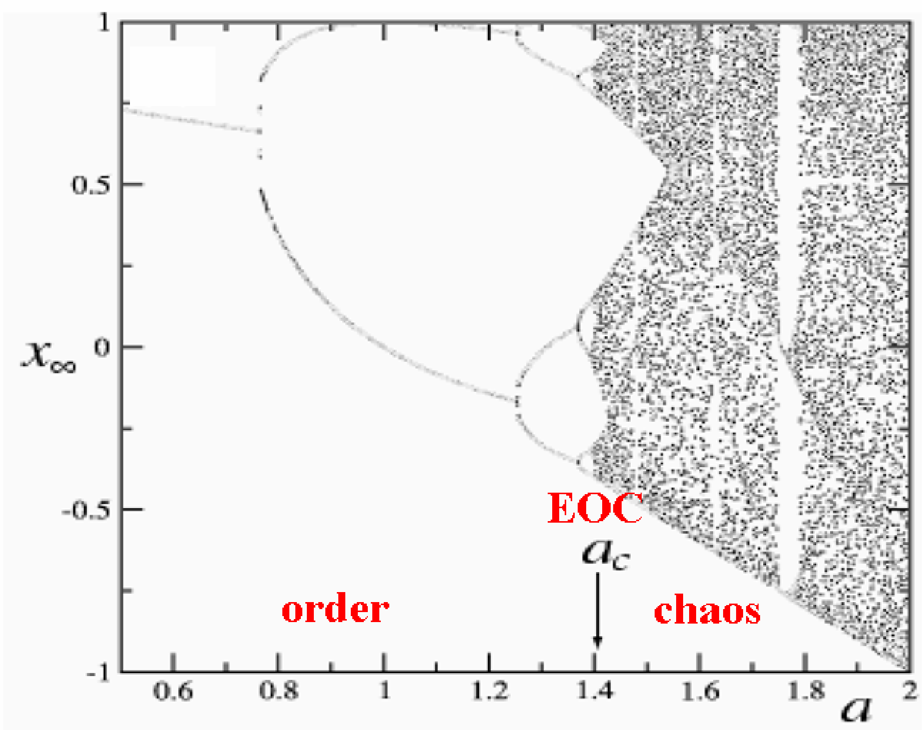

Fig. 1: Attractor of the logistic map as a function of a. The edge of chaos (EOC) is at the critical value $a_c = 1.40115519809...$ (adapted from (Tsallis et al. 2002)).

Results

We have studied the model of long range coupled logistic maps of 1D and 2D lattice with different values of logistic map parameter *a*, coupling strength *B* and the power of long range coupling *C*. The typical results (spatial and temporal dynamics, probability distributions and the degree of synchronization) are shown on Figs (2-5). The instantaneous amplitude of the logistic map is represented by color gradient, where black corresponds to the minimum (-1) and white to the maximum (+1). We always started with random initial distribution.

The model exhibits periodic 1D (Fig. 2) and 2D (Fig. 3) as well as chaotic (Fig. 4) dynamics and pattern formation (Fig. 5).

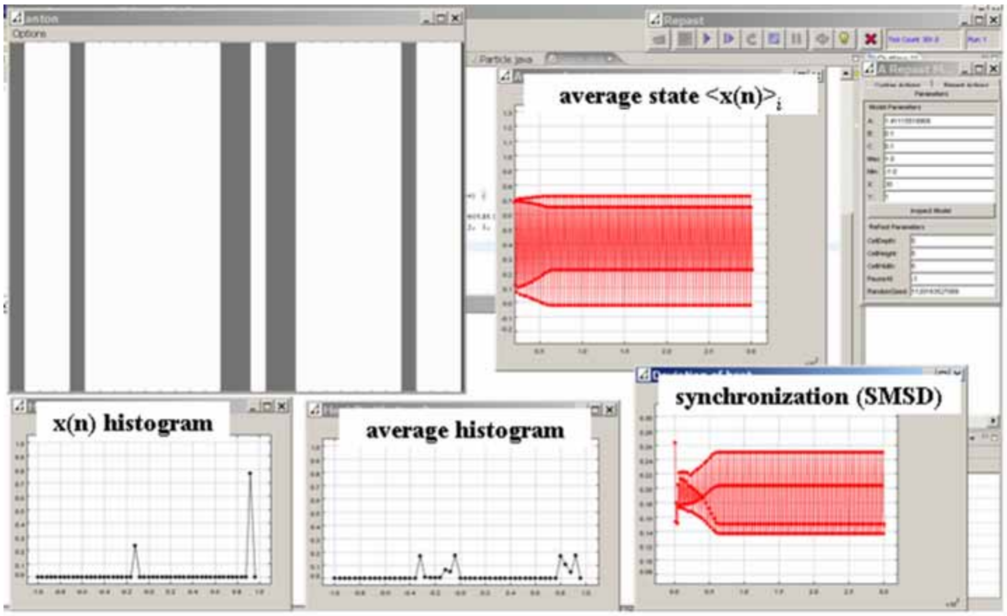

Fig. 2: Periodic dynamics on 30x1 1D grid with *a=1.41115518909*, *B=0.1* and *C=0.1*.

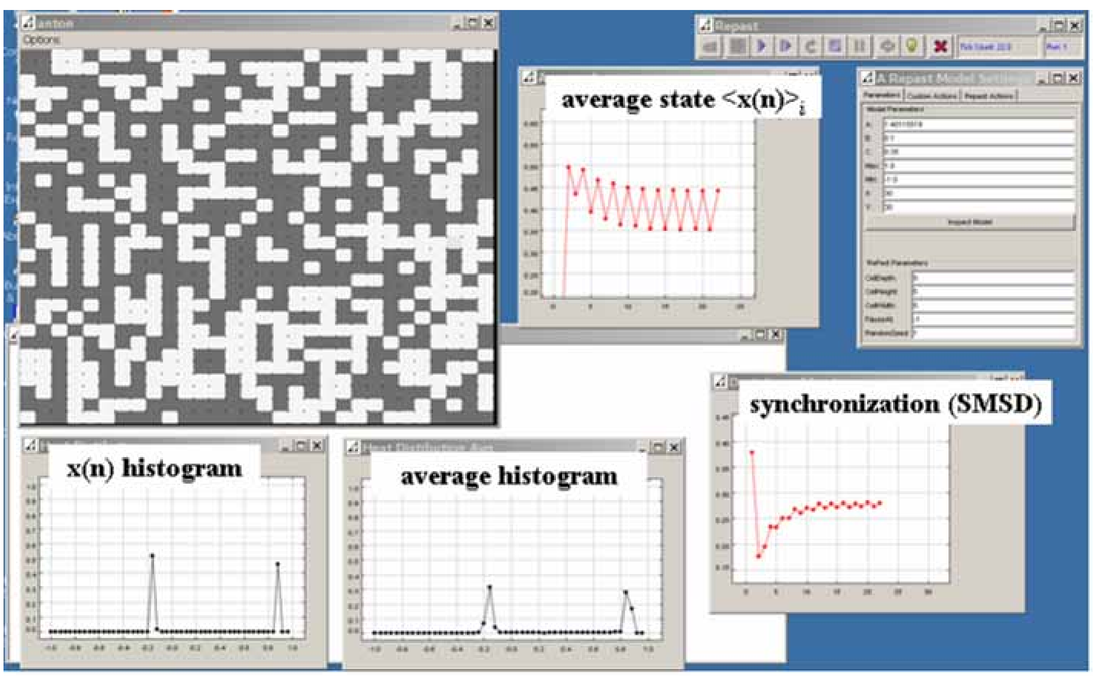

Fig. 3: Periodic dynamics on 30x30 2D grid with *a=1.40115519*, *B=0.1* and *C=0.35*.

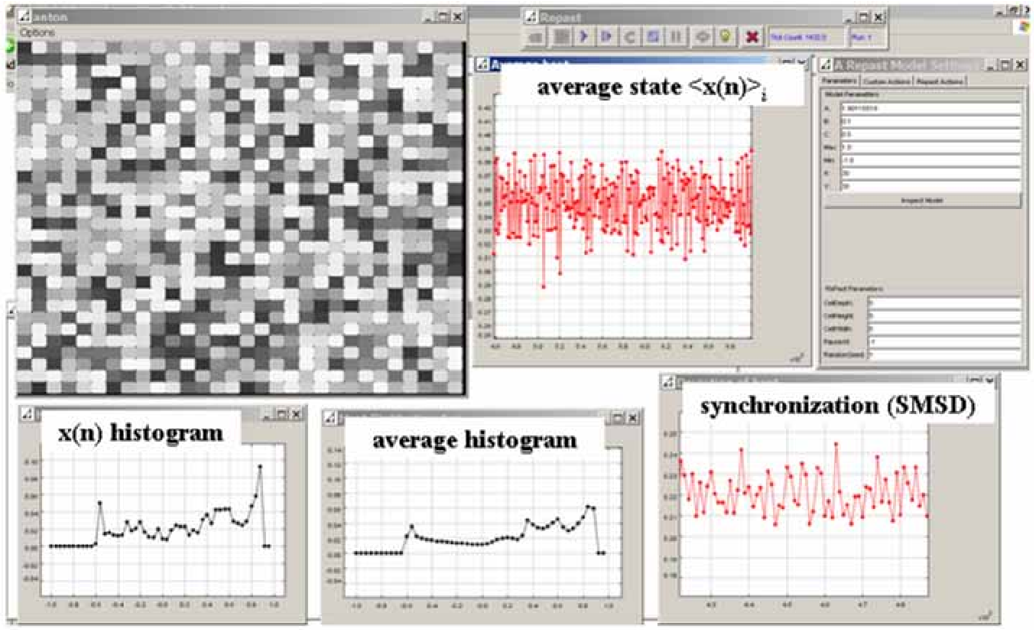

Fig. 4: Chaotic dynamics on 30x30 2D grid with *a=1.90115519*, *B=0.1* and *C=0.5*.

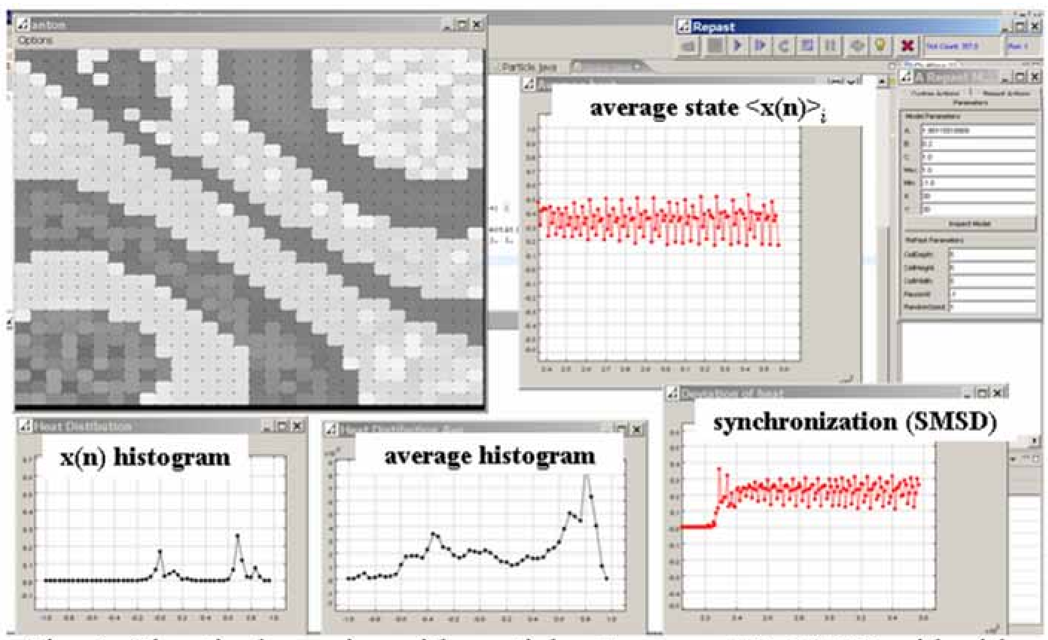

Fig. 5: Chaotic dynamics with spatial patterns on 30x30 2D grid with *a=1.90115518909*, *B=0.2* and *C=1.0* (in order to obtain the patterns, the parameters of the model have been changed several times during the simulation).

Future Work

Currently we are working on our model in several different directions (A. Burykin and B. Adamcsek, in preparation). First of all, we are studying the dynamics of the model at the EOC (with the critical value a_c of the logistic map parameter) in order to find out if the system is better characterized by the Tsallis entropy S_q rather than by the traditional Boltzmann-Gibbs entropy (see (Anteneodo et al. 1998; Tamarit et al. 2000; Tsallis et al. 2002; Tsallis et al. 2003; Pluchino et al. 2004)).

We are also investigating the effect of noise and adaptation of the GCM. More specifically, we are studying long-range coupled self-adjusting logistic maps in the presence of noise. The dynamics of the single logistic map is described by

$$x_{i+1} = 1 - a_i x_i^2 + \xi(i),$$

where $\xi(i)$ is the Gaussian noise and the logistic map parameter now is a function of x:

$$a_{i+1} = a_i + \varepsilon f_n$$

where f_n is low pass filter of the sequence $[x_n]$ of n values of the logistic map and is ε the sensitivity parameter (see (Melby et al. 2000)). We are interested in the effect of long-range coupling on such dynamics.

The most interesting direction is to implement the real ABM features, namely, to use "moving agents" (agents that can move inside the lattice "world"). This approach is very similar to the coupled map gas dynamics (Shibata et al. 2002). However one can expect that the ABM will explore more complex behavior than the simple Newtonian-like dynamics of the map gas.

It is also interesting to study the effect of the periodic boundary conditions and the use of different, more complex (e.g. random, scale-free, etc) networks on the model behavior.

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The program source code, movie of pattern dynamics of Fig. 5 and the Power Point slides of the CSSS05 talk are available on the author website http://futura.usc.edu/anton/complex.htm.

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